RADIATION OF RELATIVISTIC CHARGED PARTICLES IN A SYSTEM WITH ONE DIMENSIONAL RANDOMNESS

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Abstract

Radiation of relativistic charged particles in a system of randomly spaced plates is considered in the paper. It is shown that for large number of plates ($N \gg 1$), in the wavelength range $\lambda \ll l \ll L$ (where l is the photon mean free path and L is the system characteristic size) and for angles $|\cos\theta| \gg (\lambda/2\pi l)^{1/3}$, pseudophoton diffusion represents the major mechanism of radiation. Total intensity of radiation is investigated and its strong dependence on the particle energy and plate number is obtained.

1   Introduction

Charged particle radiation in layered media has been considered in many papers (see e.g. [1] and [2] and references therein). The interest in these systems is caused by the possibility of their use as high energy particle detectors [2].Detecting properties of these systems are based on the transition radiation. Transition radiation originating in such systems can be explained in the following way (see [1] and [2]). A charge moving in a medium creates an electromagnetic field (a pseudophoton), which is scattered by the inhomogeneities of dielectric permittivity and converted into radiation. The key problem is account correctly for the scattering of pseudophotons on the inhomogeneities.

In earlier articles (see, for example , [2]) which have addressed to the problem of radiation of relativistic charged particles in a system of plates embedded in a homogeneous medium the reflection of the electromagnetic field by an individual plate is neglected. However from experience with three dimensional random media [3] and [4] we know that the multiple scattering of electromagnetic fields plays an essential role. This role is particularly important in the optical region which we have mainly in mind.

In the present paper we consider multiple scattering effects (taking into account also reflection) when a charged particle radiates passing through a one-dimensional random medium. Such media can be, in particular, those systems in which the plates are randomly spaced in a homogeneous medium.

It turns out that multiple scattering of the pseudophoton leads to its diffusion is the dominant in the medium and this diffusion contribution to the radiation intensity. The diffusion contribution leads to a strong dependence of the radiation intensity on particle energy and plate number, a fact that is important for the detecting properties of the system. Note that the diffusion contribution is absent in an ordered stack of plates.

2   Formulation of the Problem

The system considered in the paper consits of a stack of plates randomly spaced in a

homogeneous medium. Let the plates fill the regions $z_i - a/2 < z < z_i + a/2$ (where a is the plate thickness and z_i are random coordinates). The dielectric permittivity of the system may be represented in the following form:

$$\varepsilon(z,\omega) = \varepsilon_0(\omega) + \sum_i [b(\omega) - \varepsilon_0(\omega)] [|\Theta(z - z_i - a/2) - \Theta(z - z_i + a/2)|], \qquad (1)$$

where $\varepsilon_0(\omega)$ and $b(\omega)$ are respectively dielectric permittivities of the homogeneous medium and of the plates, and Θ is a step function. It is convenient to represent the dielectric permittivity as a sum of average and fluctuating parts:

$$\varepsilon(z,\omega) = \varepsilon + \varepsilon_r(z,\omega), \qquad < \varepsilon_r(z,\omega) >= 0, \qquad (2)$$

where $\varepsilon =< \varepsilon(z,\omega) >$, $\varepsilon_r \ll \varepsilon$ and averaging over the random coordinates of plates is determined as follows

$$< f(z,\omega) >= \int \prod_i \frac{dz_i}{L_z} f(z, z_i, \omega), \qquad (3)$$

where L_z is the system size in the z-direction. In the frequency domain, Maxwell's equations have the following form:

$$\vec{\nabla} \times \vec{E}(\vec{r},\omega) = \frac{i\omega}{c} \vec{B}(\vec{r},\omega), \quad \vec{\nabla} \times \vec{B}(\vec{r},\omega) = \frac{4\pi e}{c} \frac{\vec{v}}{v} \delta(x)\delta(y)e^{i\omega z/v} - \frac{i\omega}{c} \vec{D}(\vec{r},\omega)$$

$$\vec{\nabla} \cdot \vec{D}(\vec{r},\omega) = \frac{4\pi e}{v} \delta(x)\delta(y)e^{i\omega z/v} \quad , \vec{\nabla} \cdot \vec{B}(\vec{r},\omega) = 0, \quad \vec{D}(\vec{r},\omega) = \varepsilon(z,\omega)\vec{E}(\vec{r},\omega) \,(4)$$

Here $\vec{v} \parallel \hat{z}$ is the velocity of the particle. For convenience we introduce the potentials of electromagnetic field.

$$\vec{E}(\vec{r},\omega) = \frac{i\omega}{c} \vec{A}(\vec{r},\omega) - \nabla\varphi(\vec{r},\omega) \qquad (5)$$

Using (4) and (5), we obtain the equation for $\vec{A}(\vec{r},\omega)$

$$\nabla^2 \vec{A} + \frac{\omega^2}{c^2} \vec{A}(\vec{r},\omega)\varepsilon(\vec{r},\omega) - \nabla\left[\vec{\nabla} \cdot \vec{A} - \frac{i\omega}{c}\varepsilon(\vec{r},\omega)\varphi(\vec{r},\omega)\right] = \vec{j}(\vec{r},\omega), \qquad (6)$$

where $\vec{j}(\vec{r},\omega)$ is the Fourier transform of the current of the charged particle

$$\vec{j}(\vec{r},\omega) = -\frac{4\pi e}{c} \frac{\vec{v}}{v} \delta(x)\delta(y)e^{i\omega z/v} \qquad (7)$$

Imposing the Lorentz gauge condition on the potentials, we finally obtain finally

$$\vec{\nabla} \cdot \vec{A} - \frac{i\omega}{c}\varepsilon(\vec{r},\omega)\varphi(\vec{r},\omega) = 0; \quad \nabla^2 \vec{A} + \frac{\omega^2}{c^2}\varepsilon(\vec{r},\omega)\vec{A}(\vec{r},\omega) = \vec{j}(\vec{r},\omega) \qquad (8)$$

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It follows from the symmetry of the problem that the vector potential $\vec{A}$ is directed along the z: $A_i = \delta_{\hat{z}i} A(\vec{r}, \omega)$.

3 Radiation Tensor

As usual, we decompose the electric field into two parts, $\vec{E} = \vec{E}_0 + \vec{E}_r$. Here $\vec{E}_0$ is the electric field of the charged particle moving in homogeneous medium with dielectric permittivity ε, and E_r is the radiation field caused by fluctuations in the dielectric permittivity. We define the radiation tensor as follows

$$I_{ij}(\vec{R}) = E_{ri}(\vec{R}) E_{rj}^*(\vec{R}) \tag{9}$$

Here $\vec{R}$ is the radius-vector to the observation point, which is far from the system, $R \gg L$. The vector potential is decomposed in a similar way: $\vec{A} = \vec{A}_0 + \vec{A}_r$, where $\vec{A}_0$ and $\vec{A}_r$, as follows from (refBC) and (8), satisfy the equations

$$\nabla^2 \vec{A}_0 + \frac{\omega^2}{c^2} \varepsilon \vec{A}_0 = \vec{j}(\vec{r}, \omega)$$
$$\nabla^2 \vec{A}_r + \frac{\omega^2}{c^2} \varepsilon \vec{A}_r + \frac{\omega^2}{c^2} \varepsilon_r \vec{A}_r = -\frac{\omega^2}{c^2} \varepsilon_r \vec{A}_0 \tag{10}$$

It is convenient to express the radiation intensity in terms of the radiation potential $\vec{A}_r$

$$< I_{ij}(\vec{R}) > = \frac{\omega^2}{c^2} \delta_{\hat{z}i} \delta_{\hat{z}j} < A_r(\vec{R}, \omega) A_r^*(\vec{R}, \omega) > + \frac{\delta_{\hat{z}i}}{\varepsilon} < A_r(\vec{R}, \omega) \frac{\partial^2}{\partial R_j \partial z} A_r^*(\vec{R}, \omega) >$$
$$+ \frac{\delta_{\hat{z}j}}{\varepsilon} < A_r^*(\vec{R}, \omega) \frac{\partial^2}{\partial R_i \partial z} A_r(\vec{R}, \omega) > + \frac{c^2}{\omega^2 \varepsilon^2} < \frac{\partial^2}{\partial R_i \partial z} A_r(\vec{R}, \omega) \frac{\partial^2}{\partial R_j \partial z} A_r^*(\vec{R}, \omega) > \tag{11}$$

In obtaining (11) we assumed that the fluctuations of dielectric permittivity are much smaller than its mean value $\varepsilon_r \ll \varepsilon$.

To carry out averaging over the random coordinates of plates, we express the radiation potential A_r in terms of the Green's function of the equation (10)

$$A_r(\vec{R}) = -\frac{\omega^2}{c^2} \int \varepsilon_r(\vec{r}) A_0(\vec{r}) G(\vec{R}, \vec{r}) d\vec{r}$$
$$\left[\nabla^2 + k^2 + \frac{\omega^2}{c^2} \varepsilon_r(z) \right] G(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}'), \tag{12}$$

where $k = \omega \sqrt{\varepsilon}/c$.

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4 Green's Function

The bare Green's function of equation (12) satisfies the equation

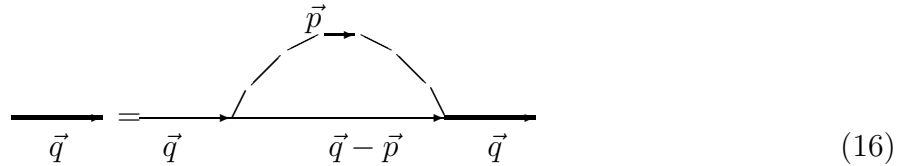$$\left[\nabla^2 + k^2 + i\delta\right] G_0(\vec{r} - \vec{r'}) = \delta(\vec{r} - \vec{r'}) \tag{13}$$

where $i\delta$, as usual, is an infinitizimal imaginary term. Solution in the momentum representation has the form:

$$G_0(\vec{q}) = \frac{1}{k^2 - q^2 + i\delta} \tag{14}$$

In the coordinate representation, one has

$$G_0(r) = -\frac{1}{4\pi r} e^{ikr} \tag{15}$$

To perform the averaging, we use the impurity-diagram method [5]. Summing the diagrams in the ladder approximation, we obtain Dyson's equation for the average Green's function

$$\tag{16}$$

The dotted line denotes the correlation function of the one-dimensional random field

$$- - - - = B(\vec{p}) = (2\pi)^2 \delta(\vec{p}_\rho) B(|p_z|)$$
$$B(|z - z'|) = \frac{\omega^4}{c^4} < \varepsilon_r(z)\varepsilon_r(z') > \tag{17}$$

where $\vec{p}_\rho$ is the transverse component of $\vec{p}$. The solution of equation (16) can be represented in following form:

$$G(\vec{q}) = \frac{1}{G_0^{-1}(\vec{q}) - \int \frac{d\vec{p}}{(2\pi)^3} B(\vec{p}) G_0(\vec{q} - \vec{p})} \tag{18}$$

Using expression (14), we obtain for the averaged Green's function the following expression:

$$G(\vec{q}) = \frac{1}{k^2 - q^2 + i\mathrm{Im}\Sigma(\vec{q})}, \tag{19}$$

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in which the imaginary part $\mathrm{Im}\Sigma$ of the self-energy is determined by Ward's identity

$$\mathrm{Im}\Sigma(\vec{q}) = \int \frac{d\vec{p}}{(2\pi)^3} B(\vec{p}) \mathrm{Im} G_0(\vec{q} - \vec{p}) = \frac{1}{4\sqrt{k^2 - q_\rho^2}}$$
$$\left[B(|q_z - \sqrt{k^2 - q_\rho^2}|) + B(|q_z + \sqrt{k^2 - q_\rho^2}|) \right], \quad |\vec{q}_\rho| < k \tag{20}$$

The decay length of pseudophoton in the z direction is determined by the imaginary part of the Green's function, in the following way (see., e.g. [6])

$$l(\vec{q}) = \frac{\sqrt{k^2 - q_\rho^2}}{\mathrm{Im}\Sigma(\vec{q})} \tag{21}$$

As one could expect, the decay length depends on the pseudophoton momentum direction. In the case where the momentum is directed along z, one obtains from (21) and (20)

$$l(\theta = 0) = \frac{4k^2}{B(0) + B(2k)} \tag{22}$$

We shall call this quantity the pseudophoton mean free path.

Using (1), (2) and (17) one finds for correlation function

$$B(q_z) = \frac{4(b - \varepsilon)^2 n \sin^2 q_z a/2}{q_z^2} \frac{\omega^4}{c^4} \tag{23}$$

Here $n = N/L_z$ is concentration of plates in the system. From (23) it follows that $B(0) = \omega^4/c^4 \times (b - \varepsilon)^2 n a^2$. On the other hand, when $ka \gg 1$, $B(2k)/B(0) \sim 1/(ka)^2 \ll 1$. Therefore the photon mean free path is

$$l \equiv l(\theta = 0) \approx \begin{cases} 4k^2/B(0), & ka \gg 1 \\ 2k^2/B(0), & ka \ll 1 \end{cases} \tag{24}$$

The calculation carried out above is correct only in the weak scattering regime, when $\frac{\mathrm{Im}\Sigma(\vec{q})}{k^2 - q_\rho^2} \ll 1$. Using (20) we obtain

$$\frac{B(0) + B(2k|\cos\theta|)}{4k^3|\cos\theta|^3} \ll 1 \tag{25}$$

From (25) it follows that at $\theta \approx \pi/2$ the condition of weak scattering is not satisfied. This is natural, because in this case the pseudophoton moves parallel to the plates. Taking $\theta = \pi/2 - \delta$ and using (22) and (25), one has $\delta \gg (1/kl)^{1/3}$.

5 Radiation Intensity in the Single Scattering Approximation

Substitution of (12) into (11) gives the following expression for radiation tensor

$$I_{ij}(\vec{R}) = \delta_{\hat{z}i}\delta_{\hat{z}j}\frac{\omega^6}{c^6}\int d\vec{r}d\vec{r}'A_0(\vec{r})A_0^*(\vec{r}') < \varepsilon_r(\vec{r})\varepsilon_r(\vec{r}')G(\vec{R},\vec{r})G^*(\vec{r}',\vec{R}) >$$

$$+\frac{\omega^2}{c^2}\frac{1}{\varepsilon^2}\int d\vec{r}d\vec{r}'A_0(\vec{r})A_0^*(\vec{r}') < \varepsilon_r(\vec{r})\varepsilon_r(\vec{r}')\frac{\partial^2}{\partial R_i\partial z}G(\vec{R},\vec{r})\frac{\partial^2}{\partial R_j\partial z}G^*(\vec{r}',\vec{R}) >$$

$$+\delta_{\hat{z}j}\frac{\omega^4}{c^4\varepsilon}\int d\vec{r}d\vec{r}'A_0(\vec{r})A_0^*(\vec{r}') < \varepsilon_r(\vec{r})\varepsilon_r(\vec{r}')G^*(\vec{r},\vec{R})\frac{\partial^2}{\partial R_i\partial z}G(R,r) >$$

$$+\delta_{\hat{z}i}\frac{\omega^4}{c^4\varepsilon}\int d\vec{r}d\vec{r}'A_0(\vec{r})A_0^*(\vec{r}') < \varepsilon_r(\vec{r})\varepsilon_r(\vec{r}')G(\vec{R},\vec{r})\frac{\partial^2}{\partial R_j\partial z}G^*(\vec{r},\vec{R}) > \tag{26}$$

In the single scattering approximation, we substitute the Green's functions appearing in (26) by bare one (15) functions. Since the observation point $\vec{R}$ is far from radiating system, one finds using (15) the following useful relations

$$G_0(\vec{R},\vec{r}) \approx -\frac{1}{4\pi R}e^{ik(R-\vec{n}\vec{r})}, \quad \frac{\partial^2 G_0(\vec{R},\vec{r})}{\partial R_i\partial z} \approx \frac{k^2 n_i n_z}{4\pi R}e^{ik(R-\vec{n}\vec{r})}, \quad R \gg r \tag{27}$$

Here $\vec{n}$ is the unit vector in the direction of observation point $\vec{R}$. Inserting (27) into (26) and using (17), for the radiation tensor we find

$$I_{ij}^0(\vec{R}) = \frac{\omega^2}{c^2}\frac{1}{16\pi^2 R^2}\int d\vec{r}d\vec{r}'e^{ik\vec{n}(\vec{r}-\vec{r}')}B(|z-z'|)A_0(\vec{r})A_0^*(\vec{r}')$$

$$\left[\delta_{\hat{z}i}\delta_{\hat{z}j} - \delta_{\hat{z}i}n_j n_z - \delta_{\hat{z}j}n_i n_z + n_i n_j n_z^2\right] \tag{28}$$

By solving (10), we easily obtain

$$A_0(\vec{q}) = -\frac{8\pi^2 e}{c}\frac{\delta(q_z - \omega/v)}{k^2 - q^2} \tag{29}$$

Using (29) in (28) and integrating, we find the radiation intensity $I(\vec{n}) = \frac{c}{2}R^2 I_{ii}(\vec{R})$ in the single scattering approximation:

$$I^0(\vec{n}) = \frac{\pi e^2}{c}\delta(0)\frac{B(|k_0 - kn_z|)n_\rho^2}{[k^2 n_z^2 - k_0^2]^2}\frac{\omega^2}{c^2} \tag{30}$$

Here $k_0 = \omega/v$ while the δ-type singularity of (30) is caused by the infinite path of the charged particle in the medium. If one takes into account the finite size of the system, $\delta(0)$
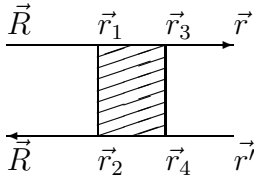
must be replaced by $L_z/2\pi$. To analyse the angular dependence of (30), it is convenient to represent it in the form
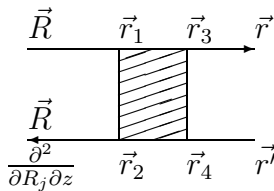
$$I^0(\theta) = \frac{e^2}{2c} \frac{L_z B(|k_0 - k\cos\theta|)\sin^2\theta}{\left[\gamma^{-2} + \sin^2\theta\, k^2/k_0^2\right]^2} \frac{\omega^2}{k_0^4 c^2} \tag{31}$$
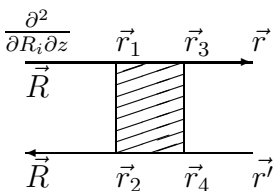
Here $\gamma = (1 - \varepsilon v^2/c^2)^{-1/2}$ is the Lorentz factor of the particle. Note some features of the expression (31): For relativistic energies ($\gamma \gg 1, k_0 \to k$), the radiation intensity in the forward direction, for short waves $ka \gg 1$, is significantly higher than in the backward direction. The maximum lies in the range of angles $\theta \sim \gamma^{-1}$. This result is consistent with the results of [1] and [2]. Since $B \sim n$, the radiation intensity in this approximation, as one should expect, is proportional to the total number N of plates in the system.

6   Diffusion Contribution to the Radiation Intensity

In the diffusion approximation, the averages appearing in (26) are determined by the following diagrams

$$< \frac{\partial^2}{\partial R_i \partial z} G(\vec{R}, \vec{r}) \frac{\partial^2}{\partial R_j \partial z} G^*(\vec{r}', \vec{R}) >^D = \quad (32)$$

Here the filled rectangle corresponds to the diffusion propagator

$$P(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = \quad = \sum \quad (33)$$

Using (26),(32) and (33), we obtain the following expression for the diffusion contribution

$$I_{ij}^D(\vec{R}) = \frac{k^2}{16\pi^2 R^2 \varepsilon} \int d\vec{r} d\vec{r}' B(r - r') A_0(\vec{r}) A_0^*(\vec{r}') \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4$$
$$e^{-ik\vec{n}(\vec{r}_1 - \vec{r}_2)} P(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) G(\vec{r}_3, \vec{r}) G^*(\vec{r}, \vec{r}_4)$$
$$\left[\delta_{\hat{z}i} \delta_{\hat{z}j} + n_i n_j n_z^2 - \delta_{\hat{z}i} n_j n_z - \delta_{\hat{z}j} n_i n_z \right] \quad (34)$$

The diffusion propagator P which appears in (34) is found similarly to the three dimensional case [4]. It follows from (33) that $P(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)$ can be represented in form

$$P(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = B(\vec{r}_1 - \vec{r}_2) B(\vec{r}_3 - \vec{r}_4) P(\vec{R}', \vec{r}_1 - \vec{r}_2, \vec{r}_3 - \vec{r}_4) \quad (35)$$

where $\vec{R}' = \frac{1}{2}(\vec{r}_3 + \vec{r}_4 - \vec{r}_1 - \vec{r}_2)$ and P satisfies the equation

$$\int \frac{d\vec{p}}{(2\pi)^3} \left[1 - \int \frac{d\vec{q}}{(2\pi)^3} f(\vec{q}, \vec{K}) B(\vec{p} - \vec{q}) \right] P(\vec{K}, \vec{p}, \vec{q}') = f(\vec{q}', \vec{K}) \quad (36)$$

Here

$$f(\vec{q}, \vec{K}) = G(\vec{q} + \vec{K}/2) G^*(\vec{q} - \vec{K}/2) \quad (37)$$

As it will be seen further, one has to know P when $\vec{K} \to 0$. In this limit, the diffusion propagator has the form [4]

$$P(\vec{K} \to 0, \vec{p}, \vec{q}) = \frac{\text{Im} G(\vec{p}) \text{Im} G(\vec{q})}{\text{Im} \Sigma(\vec{q})} A(\vec{K}) \quad (38)$$

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where

$$A(\vec{K}) = \left[3 \int \frac{(\vec{q}\vec{K})^2 \mathrm{Im} G(\vec{q})}{\mathrm{Im}^2 \Sigma(\vec{q})} \frac{d\vec{q}}{(2\pi)^3} \right]^{-1} \tag{39}$$

Choosing $\vec{K} \parallel \hat{z}$ and using (20), we obtain

$$A(K) = \left[\frac{6K^2 k^5}{\pi} \int\limits_{-1}^{1} \frac{dx x^4}{[B(0) + B(2k|x|)]^2} \right]^{-1} \tag{40}$$

Here we have changed variables while integrating over the angles. It follows from the form of correlation function (23) that the main contribution into the integral(40) is given by the values of x close to unity (the corresponding angles are close to zero). Taking into account this fact, for $A(K)$, we have approximately

$$A(K) = \frac{1}{k} \frac{20\pi}{3K^2 l^2}, \tag{41}$$

where $l = 4k^2/B(0)$ is the pseudophoton's mean free path. In the expression for radiation intensity it is convenient to turn to new variables of integration

$$\vec{R}' = \frac{1}{2}(\vec{r}_3 + \vec{r}_4 - \vec{r}_1 - \vec{r}_2), \quad \vec{x}_1 = \vec{r}_1 - \vec{r}_2, \quad \vec{x}_2 = \vec{r}_3 - \vec{r}_4, \quad \vec{r}_4 \equiv \vec{r}_4, \tag{42}$$

which gives

$$I_{ij}^D(\vec{R}) = \frac{k^2}{16\pi^2 R^2 \varepsilon} \left(\delta_{\hat{z}i} \delta_{\hat{z}j} + n_i n_j n_z^2 - \delta_{\hat{z}i} n_j n_z - \delta_{\hat{z}j} n_i n_z \right) D, \tag{43}$$

where D is given by the expression

$$D = \int d\vec{r} d\vec{r}' d\vec{R}' d\vec{x}_1 d\vec{x}_2 d\vec{r}_4 A_0(\vec{r}) B(r - r') A_0^*(\vec{r}') e^{-ik\vec{n}\vec{x}_1}$$
$$B(x_1) B(x_2) P(\vec{R}', \vec{x}_1, \vec{x}_2) G(\vec{x}_2 + \vec{r}_4 - \vec{r}) G^*(\vec{r}' - \vec{r}_4) \tag{44}$$

In the Fourier representation (44) has the following form

$$D = \int \frac{d\vec{q}_1 d\vec{q}_2 d\vec{q}_3 d\vec{q}_4}{(2\pi)^{12}} |A_0(\vec{q}_1)|^2 B(\vec{q}_2) B(\vec{q}_3) B(\vec{q}_4)$$
$$P(K' \to 0, -\vec{q}_3 - k\vec{n}, \vec{q}_1 + \vec{q}_2 + \vec{q}_4) |G(\vec{q}_1 + \vec{q}_2)|^2 \tag{45}$$

Substituting (38) into (45) and integrating (using the Ward identity (20)), we shall obtain

$$D = A(K) \mathrm{Im} \Sigma(k\vec{n}) \int \frac{d\vec{q}}{(2\pi)^3} \frac{B(|\sqrt{k^2 - q_\rho^2} - q_z|) + B(|\sqrt{k^2 - q_\rho^2} + q_z|)}{B(0) + B(2\sqrt{k^2 - q_\rho^2})} |A_0(\vec{q})|^2 \tag{46}$$

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Finally, we evaluate the integral over the momentum remaining in (46). Using (20) and (29) in (46), we have

$$
\begin{aligned}
D &= A(\vec{K})\mathrm{Im}\Sigma(k\vec{n})\frac{16\pi^2 e^2}{c^2}L_z \times \\
&\times \int \frac{d\vec{q}_\rho}{(2\pi)^2}\frac{1}{(k^2-k_0^2-q_\rho^2)^2}\frac{\left[B\left(|k_0+\sqrt{k^2-q_\rho^2}|\right)+B\left(|k_0-\sqrt{k^2-q_\rho^2}|\right)\right]}{B(0)+B(2\sqrt{k^2-q_\rho^2})}
\end{aligned}
\tag{47}
$$

It follows from (47), that for relativistic energies $k_0 \to k$, the main contribution to the integral (47) is given by the values $q_\rho \to 0$. Taking into account this fact, and the fact that when $\gamma^2 \gg ak$ function B varies slowly as well as (41), we find

$$
D \approx \frac{e^2}{c^2}\frac{20\pi^2}{3K^2 l^2}L_z\frac{B(0)+B(2k|n_z|)}{k^2}\frac{1}{|n_z|}\frac{\gamma^2}{k_0^2}
\tag{48}
$$

Substituting (48) into (43) for the diffusion contribution into the radiation intensity, we obtain finally

$$
I^D(n_z) = \frac{5}{6}\frac{e^2\gamma^2}{\varepsilon c}\left(\frac{L_z}{l(\omega)}\right)^3\frac{1-n_z^2}{|n_z|}
\tag{49}
$$

In deriving (49) we substitute $1/K^2$ at $K \to 0$ by L_z^2 as usual (and also assume that $L_z \ll L_x, L_y$). Note some pecularities of the diffusion contribution. It is easy to verify that $I^D/I^0 \sim L_z^2/l^2 \gg 1$. This means that for $k|\cos\theta|^3 l \gg 1$ and $l \ll L_z$ the diffusion contribution is the major one. As one should expect, the backward and forward intensities are equal to each other. Note that with an accuracy of unimportant numerical coefficients the formula (49) is correct both for short $ka \gg 1$ and for long $ka \ll 1$ waves. All information on randomness is contained in the mean free path $l(\omega)$. In the next section we shall specify the form of $l(\omega)$ in particular cases.

7   Pseudophoton Mean Free Path

The pseudophoton mean free path in our theory is described by the expression (24). In the impurity diagram method [5], as usual, we don't take into account the diagrams which correspond to the situation of three or more plates at the same point. This is valid provided that $|\sqrt{b/\varepsilon}-1|ka \ll 1$ which means that for scattering of a photon on a plate,

the Born approximation is fulfilled. However it is well known [5] that the formulae are also correct in the general case provided that one employs the exact scattering amplitude instead of Born scattering amplitude. In our case this means that formula (49) is correct in the general case provided that a suitable expression is used for pseudophoton mean free path $l(\omega)$.

The photon mean free path in the medium is related to the photon transmission coefficient through a plate.

$$l(\omega) = \frac{[1 - \mathrm{Re}t(\omega)]^{-1}}{n} \tag{50}$$

where $t(\omega)$ is the photon transmission coefficient through a dielectric plate with photon momentum normal to the plate [6]

$$t(\omega) = \frac{2i\sqrt{\frac{b(\omega)}{\varepsilon(\omega)}}\exp(-ika)}{\left[\frac{b(\omega)}{\varepsilon(\omega)} + 1\right]\sin\sqrt{\frac{b(\omega)}{\varepsilon(\omega)}}ka + 2i\sqrt{\frac{b(\omega)}{\varepsilon(\omega)}}\cos\sqrt{\frac{b(\omega)}{\varepsilon(\omega)}}ka} \tag{51}$$

It follows from (49) and (50) that the maximum of spectral radiation intensity lies in the frequency region where transmission coefficient is minimal. It follows from (50) that the minimal value of $l(\omega)$ is $1/n$. Now we shall clarify the conditions under which this value is achieved. In the Born approximation $|\sqrt{b/\varepsilon} - 1|ka \ll 1$, using (51) and (50), we obtain

$$l(\omega) = \frac{2}{n\left(\sqrt{\frac{b}{\varepsilon}} - 1\right)^2 k^2 a^2} \tag{52}$$

which agrees with (24). More interesting for us is the geometrical optics region $|\sqrt{\frac{b}{\varepsilon}} - 1|ka \gg 1$. Substituting (51) into (50) and neglecting the strongly oscillating terms, we have $l(\omega) \sim 1/n$. Thus in the geometrical optics region the photon mean free path does not depend on the frequency, and radiation intensity is maximal. Integrating the spectral intensity over angles and frequencies in this region we find that the total intensity depends on the particle energy as $I^t \sim \gamma^2$. By contrast the energy dependence of the radiation intensity in typical transition radiation from a single interface in the optical region is logarithmic (see, for example,[2]). In order to find the dependence of the radiation intensity on the number of plates, note that $L_z = N/n$ and from (49) one has

$I^t \sim N^3$. One of the important conditions for the applicability of the theory is the condition $l \ll L_z$. Substituting $L_z = N/n$ and $l = 1/n$ into this condition we find a condition for plate number $N \gg 1$.

Note that we didn't take into account the absorption of photons. This is valid provided that $l \ll l_{in}$ (where l_{in} is the photon inelastic mean free path in the medium). In the theory of diffusive propagation the weak absorption ($l \ll l_{in}$) is taken into account in the following way (see, for example, [7]). If the absorption is so weak that $L_z < (l l_{in})^{1/2}$, then expression (49) remains unchanged. When $L_z > (l l_{in})^{1/2}$ one must substitute L_z^2 by $l l_{in}$ in (49)

$$I^D(n_z, \omega) = \frac{5}{6} \frac{e^2 \gamma^2}{\varepsilon c} \frac{L_z l_{in}(\omega)}{l^2(\omega)} \frac{1 - n_z^2}{|n_z|} \tag{53}$$

It follows from (53) that in this case the dependence of radiation intensity on plate number is weaker $I \sim N$.

8   Conclusions

We have considered the diffusion contribution for radiation intensity of a relativistic particle passing through a stack of randomly spaced plates. It was shown that for a large number of plates $N \gg 1$, in the wavelength region $\lambda \ll l$ and for the angles $|\cos\theta| \gg (1/kl)^{1/3}$, the diffusion contribution is the dominant one. Note that the backward and forward intensities of relativistic charged particle radiation intensity are equal, whereas in the regular stack case relativistic particle radiates mainly in the forward direction.

Now let us discuss the possible experimental realizations of our theory. For applicability of the theory the fulfilment of the following inequalities is necessary $\lambda \ll l(\lambda) \ll l_{in}, L_z$.

The transition radiation of relativistic charged particles in a stack of plates has been investigated experimentally in many papers (see, for example, [8]). Unfortunately in these papers only the X-ray region was studied. In the X-ray region the above-mentioned inequalities are not satisfied. Optical transition radiation of relativistic particles has been investigated in experimental work [9]. However in this experiment only one or two parallel plates were used. Samples in [9] were prepared by vacuum deposition of various

metallic coatings (Al, Ag, Au, Cu) on mylar foils $3.5\mu m$ thick. Note that these samples are optimal for our goals. They ensure minimal transmission due to metallic coatings and weakness of absorption due to mylar foils. So it will be interesting to investigate experimentally the optical ($\lambda \sim 2000A^0 - 6000A^0$) transition radiation of relativistic ($\gamma \sim 10^2 - 10^3$) electrons passing through a stack of such samples randomly spaced in the vacuum.

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